2.2 Change in Momentum

Figure B2.5: Owls use momentum to catch their prey.

The owl is a swift and cunning hunter that uses the physics of momentum as a "bashing force" while pouncing on its prey. Flying at a velocity of 15 m/s, a 2.1-kg owl can pack a momentum of 32 kg□m/s to stun its prey. Unfortunately, collisions between this low flying predator and vehicles sometimes occur. It is estimated that these collisions can involve forces of 14 000 N or more! How can the momentum of a relatively small bird produce such a large force in a collision with a vehicle? More importantly, how does momentum relate to force?

During the interaction of the owl with either its prey or a vehicle, the momentum of the owl changes. Recall that the equation for momentum is $\vec{p} = m\vec{v}$. So, to change the momentum of an object, the mass, the velocity, or both must change. If the mass remains constant, it is the velocity that changes—which means acceleration occurs.

In his second law, Newton stated that in order to accelerate a mass, an unbalanced force must be applied. This is often expressed with the equation $\vec{F} = m\vec{a}$.

This equation can be revised to reflect velocity. Recall that

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Therefore, $\vec{F} = m\vec{a}$

$$= m\left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right)$$

$$= \frac{m\left(\vec{v}_f - \vec{v}_i\right)}{\Delta t}$$

This version of the second law describes the relationship between a net force acting on a mass and the acceleration of the mass. A net force is what is needed to accelerate a mass. The greater the net force on a mass during a time interval, the greater its acceleration or the greater the change in velocity.

Yet another version of Newton's second law can be derived from the previous version. You know that $\vec{p} = m\vec{v}$.

$$\vec{F} = \frac{m\left(\vec{v}_f - \vec{v}_i\right)}{\Delta t}$$

$$= \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$$

$$= \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$$

$$= \frac{\Delta \vec{p}}{\Delta t}$$

This version of Newton's second law depicts the relationship between force and momentum. Simply stated, the second law says that when a force is applied to a mass over a specified time, the mass must undergo a change in momentum. The greater the force exerted on the object during the time interval, the greater the change in momentum experienced by that object.

Example Problem 2.4

A 2.1-kg barn owl flying at a velocity of 15 m/s[E] strikes head-on with the windshield of a car travelling 30 m/s[W].

a. If the time interval for the impact was 6.7×10^{-3} s, determine the force that acted on the owl.

b. Predict the effects of the collision on the owl and on the car.

Solution

a. To determine the force that acted on the owl, focus on the motion of the owl. The owl was initially travelling 15 m/s[E]. After 6.7×10^{-3} s, the owl was then travelling at the same velocity as the car, 30 m/s[W].

Let east be the positive direction.

$m = 2.1 \text{ kg}$

$\vec{v}_i = 15 \text{ m/s[E]} = +15 \text{ m/s}$

$\vec{v}_f = 30 \text{ m/s[W]} = -30 \text{ m/s}$

$\Delta t = 6.7 \times 10^{-3} \text{ s}$

$\vec{F} = ?$

$$\vec{F} = \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t}$$
$$= \frac{2.1 \text{ kg}\left[(-30 \text{ m/s}) - (+15 \text{ m/s})\right]}{6.7 \times 10^{-3} \text{ s}}$$
$$= -1.4 \times 10^{4} \text{ kg·m/s}^2$$
$$= -1.4 \times 10^{4} \text{ N}$$

The force that acted on the owl was 1.4×10^{4} N[W].

b. Unfortunately, the forces involved in this collision are large enough to likely kill the owl and cause extensive damage to the windshield.

An Average of Changing Force Values

If you applied forensic science to attempt to reconstruct the collision in Example Problem 2.4, you would be faced with a long list of details: the exact slope of the windshield, the direction of the bird's velocity relative to the car's velocity, and the list goes on.

Given all the variables, it makes sense that the force that acted on the owl was not constant throughout the entire collision. During the 67 ms (milliseconds) of the collision, the force value likely changed every millisecond. So, the force calculated in Example Problem 2.4 represents an average of all the individual forces that acted on the owl during the collision.

This same thinking will apply to virtually all collisions. So, when you state a force value for a collision, you should remember that this force represents an average of all the changing force values. Keep this in mind as you solve the next Practice questions.

Practice

4. A water balloon with a mass of 4.00 kg is dropped from a window. The balloon reaches a velocity of 31.3 m/s just before striking the ground.
   a. Determine the momentum of the balloon just before it strikes the ground.
   b. If the velocity of the balloon is 0 upon striking the ground, determine the change in momentum of the balloon.
   c. If the impact with the ground took 0.011 s, calculate the force exerted by the ground on the balloon.
5. A 2000-kg car travelling 25 m/s strikes a tree and comes to rest. If the impact took 0.23 s, determine the force exerted on the car.
6. A 500-g rubber ball is thrown at a velocity of 5.00 m/s and strikes a wall. After only 0.25 s, the ball rebounds straight back with a velocity of 4.50 m/s. Calculate the force exerted on the ball.

Factors Affecting the Change in Momentum of an Object

In motor vehicle collisions or in collisions that occur in sports, one of the objects undergoing a change in momentum is often a human. In these situations, the question becomes, "How can the momentum of a person be changed safely?" To answer this question, you need to rearrange the equation that states Newton's second law in terms of momentum.

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$ ¨ Multiply both sides by D*t*.

$$\vec{F}\Delta t = \Delta \vec{p}$$

$$\Delta \vec{p} = \vec{F}\Delta t$$

This equation indicates that the change in momentum depends on the force applied to the object and the time interval over which the force acts. In the next investigation you will explore the relationship between force and the time interval as an object experiences a change in momentum.

Investigation

Changes in Momentum

Background

Science Skills

- ✓ Performing and Recording
- ✓ Analyzing and Interpreting
- ✓ Communication and Teamwork

The momentum of a rolling marble is reduced to zero as it moves a Styrofoam cup across a surface. Since the force of friction between the cup and the surface is the force that stops the marble, the force on the marble is varied by changing the surface under the cup. The surfaces are textured paper towel, loose-leaf paper, and aluminium foil.

Before you begin collecting data, you should gently push the cup several times across each of these surfaces to get a sense of the relative size of the frictional force between the cup and each surface.

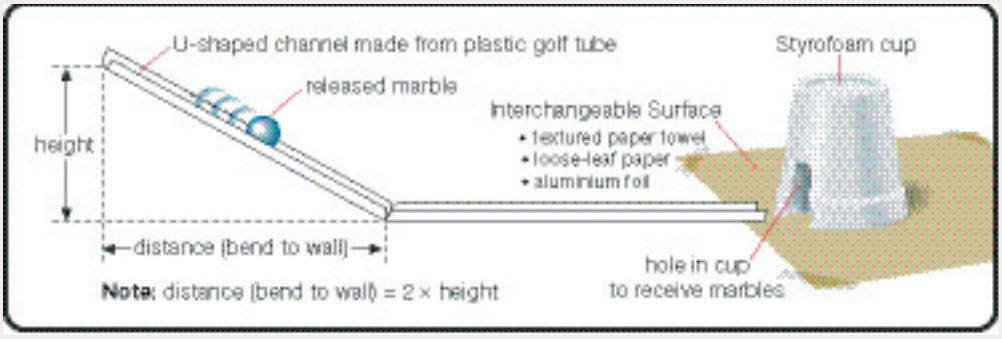
1. Based upon your observations, rank the surfaces from the greatest amount of frictional force to the least amount.

Purpose

You will determine the relationship between the force and the time interval required to change the momentum of an object.

Materials

Set up the apparatus as shown in the following diagram.



Procedure

step 1: Copy the following table into your notebook or into a spreadsheet.

Surface Under Cup	Stopping Distance (cm)					Average Stopping Distance (cm)
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	
textured paper towel						
loose-leaf paper						
aluminium foil						

step 2: Begin with the piece of textured paper towel as the first surface under the cup. Tape down the paper towel tightly to ensure there are no wrinkles. Mark the position of the cup.

step 3: Release the marble from the 30-cm mark on the inclined portion of the tube so that the marble rolls down the tube, enters the hole in the cup, and moves the cup across the paper towel.

step 4: Measure the distance the cup moves. Record your observations in the table.

step 5: Repeat steps 3 and 4 four more times. Record your observations in the appropriate place in the table.

step 6: Determine the average stopping distance. Record this value in the table.

step 7: Repeat steps 2 to 6 for the loose-leaf paper and the aluminium foil.

Analysis

2. Was the initial velocity and final velocity of the marble the same in each trial? Explain your answer.
3. Determine the relative size of the time interval required to stop the cup for each surface. Which surface provided the longest time interval? the shortest time interval?
4. Determine the relative size of the force required to stop the cup. Which surface provided the largest force? the smallest force?
5. Was the change in momentum of the marble the same in all trials? Explain your answer.

Evaluation

6. Explain the relationship between the force and the time interval to the change of momentum of an object.

Calculating the Change in Momentum

In the investigation, the change in the momentum of the marble was the same for each trial. Since the marble was released from the same height in each trial, it acquired the same velocity before hitting the inside of the cup. This meant that the marble had the same initial momentum in each trial. Further, if the marble came to rest after colliding with the cup, the final momentum in each trial was zero. Thus, the change in momentum in each trial must have been the same.

The investigation qualitatively analyzed the relationship between the force necessary to change the momentum and the time interval in which the force acts. The following equation explains this relationship:

$$\Delta \vec{p} = \vec{F}\Delta t$$

The change in momentum is provided by a large force acting over a short time interval.

$$\Delta \vec{p} = \vec{F}\Delta t$$

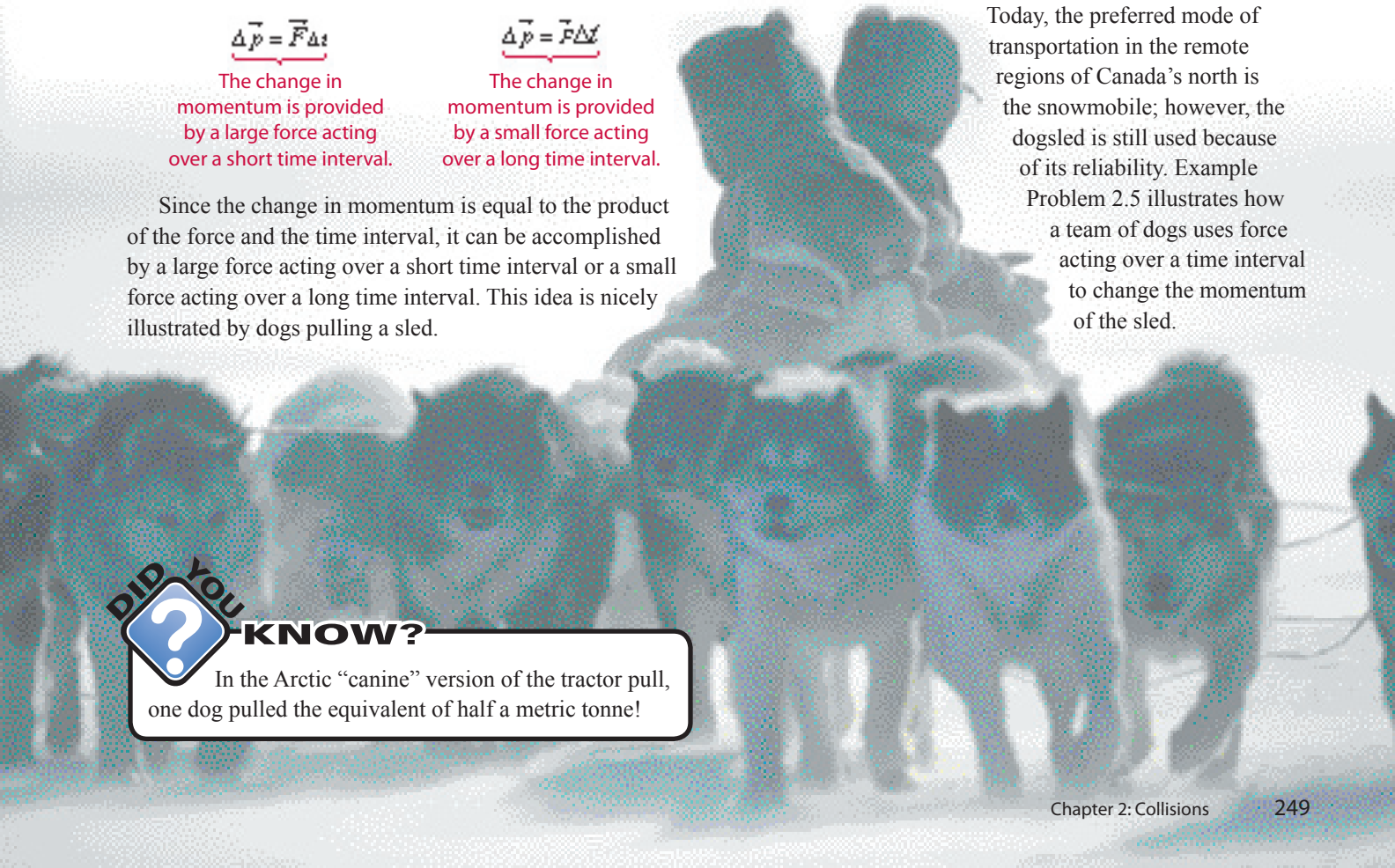
The change in momentum is provided by a small force acting over a long time interval.

Since the change in momentum is equal to the product of the force and the time interval, it can be accomplished by a large force acting over a short time interval or a small force acting over a long time interval. This idea is nicely illustrated by dogs pulling a sled.

Dogsledding is now seen as a sport featured in many modern races; but for early inhabitants of the North, it was the only means of efficient travel across the rugged landscape.

Mushers follow the ancient traditions of the First Peoples of the North, who bred dogs for their particular needs for centuries. The Malemiut Inupiat people bred a particularly hardy breed of sled dog, called the malamute. Ideally suited for winter travel for a number of reasons, malamutes are the most powerful draft animal, kilogram for kilogram, on Earth. A team of malamutes, each with an average mass of 34 kg, can easily match a pair of much larger horses.

Today, the preferred mode of transportation in the remote regions of Canada's north is the snowmobile; however, the dogsled is still used because of its reliability. Example Problem 2.5 illustrates how a team of dogs uses force acting over a time interval to change the momentum of the sled.



DID YOU KNOW?

In the Arctic "canine" version of the tractor pull, one dog pulled the equivalent of half a metric tonne!

Example Problem 2.5

Calculate the magnitude of the change in momentum in each case. Note that since this question asks for magnitude only, the vector notation is dropped.

a. A dog team consisting of two dogs is pulling a sled. If each dog provides a pulling force of 15 N, the total pulling force provided by the team is 30 N. Determine the magnitude of the change in momentum of the sled if the dogs pull for 10.0 s.

Solution

a. $\Delta p = F\Delta t$
$= (30\ \text{N})(10.0\ \text{s})$
$= 3.0 \times 10^{2}\ \text{N}\cdot\text{s}$
$= 3.0 \times 10^{2}\ \text{kg}\cdot\text{m/s}$

The change in momentum is 3.0 ¥ 10^2 kg□m/s.

b. A dog team consisting of four dogs is pulling a sled. If each dog provides a pulling force of 15 N, the total pulling force provided by the team is 60 N. Determine the magnitude of the change in momentum of the sled if the dogs pull for 10.0 s.

Solution

b. $\Delta p = F\Delta t$
$= (60\ \text{N})(10.0\ \text{s})$
$= 6.0 \times 10^{2}\ \text{N}\cdot\text{s}$
$= 6.0 \times 10^{2}\ \text{kg}\cdot\text{m/s}$

The change in momentum is 6.0 ¥ 10^2 kg□m/s.

c. A dog team consisting of two dogs is pulling a sled. If each dog provides a pulling force of 15 N, the total pulling force provided by the team is 30 N. Determine the magnitude of the change in momentum of the sled if the dogs pull for 5.0 s.

Solution

c. $\Delta p = F\Delta t$
$= (30\ \text{N})(5.0\ \text{s})$
$= 1.5 \times 10^{2}\ \text{N}\cdot\text{s}$
$= 1.5 \times 10^{2}\ \text{kg}\cdot\text{m/s}$

The change in momentum is 1.5 ¥ 10^2 kg□m/s.

d. A dog team consisting of four dogs is pulling a sled. If each dog provides a pulling force of 7.5 N, the total pulling force provided by the team is 30 N. Determine the magnitude of the change in momentum of the sled if the dogs pull for 10 s.

Solution

d. $\Delta p = F\Delta t$
$= (30\ \text{N})(10.0\ \text{s})$
$= 3.0 \times 10^{2}\ \text{N}\cdot\text{s}$
$= 3.0 \times 10^{2}\ \text{kg}\cdot\text{m/s}$

The change in momentum is 3.0 ¥ 10^2 kg□m/s.

In Example Problem 2.5, the solutions to parts a. and b. showed that over the same time interval, the change in momentum is greater with a greater force. The solutions to parts c. and d. showed that when the force acting on the object is the same, the change in momentum is greater when the time interval is longer.

Practice

7. A vehicle travelling along a highway encounters an icy patch, causing it to skid. The skid causes the vehicle to strike both a plastic pylon and a wood pylon along the side of the road. If the time interval for the impact was 0.012 s for both pylons, calculate and compare the change in momentum of the vehicle after striking a
   a. plastic pylon that exerts a resistive force of 18 N
   b. wood pylon that exerts a resistive force of 190 N
8. A child provides a constant force of 30 N by pushing on a wagon. Calculate and compare the change in momentum of the wagon if the constant force is applied for
   a. 4.0 s
   b. 8.0 s

2.2 Summary

There are two versions of Newton's second law of motion. The first version, studied in Chapter 1, states that an unbalanced force will cause a mass to accelerate. The second version, derived from the first version, states that a force acting on a mass over a specified time will cause the mass to undergo a change in momentum. This equation can be rearranged as follows:

$$\vec{F} = m\vec{a} \qquad \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \qquad \Delta \vec{p} = \vec{F}\Delta t \qquad \vec{F} = \frac{m\left(\vec{v}_f - \vec{v}_i\right)}{\Delta t}$$

As the magnitude of the force increases or as the length of time that the force acts increases, the change in momentum also increases.

Figure B2.6: To change the momentum of this car, the tree provided a large force that acted over a short time interval.

2.2 Questions

Knowledge

1. Compare two versions of Newton's second law of motion, $\vec{F} = m\vec{a}$ and $\Delta \vec{p} = \vec{F}\Delta t$, using word formulas.

2. Describe the change in momentum of an object if the
   a. amount of force acting on the object increases
   b. time interval in which a force acting on an object increases

Applying Concepts

3. Determine the change in momentum of a cart if a force of 300 N acts on the cart for 4.0 s.

4. Calculate the force required to change the momentum of a wagon by 30.0 kg·m/s in 4.5 s.

5. If a force of 65 N changes the momentum of a ball by 12 kg·m/s, determine the time interval that the force acts on the ball.

6. A force of 25 N acts on a 2.0-kg object initially at rest for 3.0 s. Determine the
   a. final momentum of the object
   b. final velocity of the object

7. Calculate and compare the forces required to stop the bird in each of the following cases.
   a. A 100-g sparrow flying 8.0 m/s strikes the windshield of a stationary car. The time interval of the impact for the sparrow to come to rest was 0.013 s.
   b. A 1.00-kg seagull flying 8.0 m/s strikes the windshield of a stationary car. The time interval of the impact for the seagull to come to rest was 0.050 s.